MTH 203: Introduction to Groups and Symmetry

Homework V

(Due 12/10/2022)

Problems for submission

1. Use the Second isomorphism theorem to show that given positive integers m, n

$$mn = \gcd(m, n)\text{lcm}(m, n).$$

2. Show that every proper subgroup of a group of order 8 is abelian.

3. Let a finite group G be a direct product of the groups $\mathbb{Z}_{n_i}$, for $1 \leq i \leq k$. Show that if $g = (g_1, g_2, \ldots, g_k) \in G$, then

$$o(g) = \text{lcm}(o(g_1), o(g_2), \ldots o(g_k)).$$

[Hint: $o(g)$ is the smallest integer r such that $g_i^r = 1$, for all i.]

4. Up to isomorphism, classify all abelian groups of orders 64 and 100.

Problems for practice

1. Establish the assertions in 3.3 (vii), 4.1 (iii) - (v), and 4.2 (viii) of the Lesson Plan.

2. Given a prime p and an integer $k \geq 1$, show that there exists an abelian group of order p^k in which every nontrivial element is of order p.

3. Given a prime p and an integer $k \geq 2$, classify all abelian groups of order p^k.

4. Show that up to isomorphism and rearrangement the direct product of k groups G_i, for $1 \leq i \leq k$, is unique.